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Boson-fermion bound states in two dimensional QCD

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Abstract

We derive the boson-fermion bound state equation in a two dimensional gauge theory in the large- N_c limit. We analyze the properties of this equation and in particular, find that the mass trajectory is linear with respect to the bound state level for the higher mass states.

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1. Introduction

It is quite common in particle physics to encounter the possibility of boson–fermion bound states. This possibility often arises in technicolor theories, especially when we have supersymmetry. Also, it should not be forgotten that in order to understand the dynamics of the standard electroweak model, we need to consider also the region where the gauge couplings are strong and quarks and leptons are thought to be as composite particles [1]. Compositeness, in essence, is a consequence of non–perturbative phenomena and even in the most studied case, QCD, it is still considered an outstanding problem. In this regard, the large– N_c limit of QCD in two dimensions is to a large extent solvable [2] and presents an invaluable opportunity for studying QCD in a simpler situation [3][4]. Substantial amount of work has been done using this model which has yielded important physical insight into the dynamics of QCD.

In this note, we derive the boson–fermion bound state equation in the large– N_c limit of two dimensional QCD analogous to the fermion–fermion [2] and the boson–boson bound state equations [4][5] derived previously. The properties of this equation will be examined and it is shown that the higher mass bound states follow a trajectory linear with respect to the bound state level.

The Lagrangian of the model is

$$-\mathcal{L} = \frac{1}{4}\text{tr}(F_{\mu\nu}^2) + \sum_a \bar{\psi}_a (\not{D} + m_{(f)a}) \psi_a + \sum_a \left(|D\phi_a|^2 + m_{(b)a}^2 |\phi_a|^2 \right) \quad (1.1)$$

a denotes the flavor index and $(f), (b)$ indices are used to indicate quantities pertaining respectively to fermions and bosons. The space–like metric $(-+)$ will be used in this work. We choose the gauge group to be $SU(N_c)$ and the matter fields ϕ_a, ψ_a to belong to the fundamental representation. We adopt the light–cone gauge $A_- = A^+ = 0$. The light–cone components are defined as $a^\pm = a_\mp \equiv 1/\sqrt{2}(a^1 \pm a^0)$.

The large– N_c limit is taken by fixing $g^2 N_c$ and the masses while taking N_c to infinity. The leading order quantum corrections to the propagators are of $\mathcal{O}(N_c^0)$ and we may solve the Schwinger–Dyson equations for the fermion and the boson propagators incorporating these contributions as, respectively,

$$\begin{aligned} S_a(p) &= \left[i\not{p} + i\frac{g^2 N_c}{2\pi} \left(\frac{\text{sign}(p_-)}{\lambda_-} - \frac{1}{p_-} \right) \gamma^+ + m_{(f)a} \right]^{-1} \\ D_a(p) &= \left[p^2 + m_{(b)a}^2 + \frac{g^2 N_c}{\pi} \frac{|p_-|}{\lambda_-} \right]^{-1} \end{aligned} \quad (1.2)$$

Here, λ_- denotes the infrared cutoff. The Schwinger–Dyson equations may be represented graphically as in fig. 1.

fig. 1 *Schwinger–Dyson equations for the self–energy part of the propagators.*

The Bethe–Salpeter equation for the bound states to leading order in $1/N_c$ may be represented diagrammatically as fig. 2 and may be obtained following 't Hooft [2];

$$\psi_{(bf)}(p, r) = g^2 D_1(p) S_2(p - r) \gamma^+ \int \frac{d^2 k}{(2\pi)^2} \frac{(2p_- + k_-)}{k_-^2} \psi_{(bf)}(k + p, r) \quad (1.3)$$

fig. 2 *Bethe–Salpeter equations for bound states. The blobs denote the connected parts of the diagram.*

In the light–cone gauge, using $(\gamma^+)^2 = 0$, $\psi_{(bf)}(p, r)$ is proportional to γ^- so that we define $\int dp_+ \psi_{(bf)}(p, r) \equiv \gamma^- \tilde{\varphi}_{(bf)}(p_-, r)$ to simplify the equation to

$$\mu_{(bf)}^2 \tilde{\varphi}_{(bf)}(x) = \left(\frac{\alpha_{(b)1}}{x} + \frac{\alpha_{(f)2}}{1-x} \right) \tilde{\varphi}_{(bf)}(x) - \text{P} \int_0^1 \frac{dy}{(y-x)^2} \frac{(x+y)}{2x} \tilde{\varphi}_{(bf)}(y) \quad (1.4)$$

Here we defined $x \equiv p_-/r_-$, $\alpha_{(b)i} \equiv \pi m_{(b)i}^2/g^2 N_c - 1$, similarly for $\alpha_{(f)i}$ and $\mu_{(bf)}^2$ is the bound state mass squared in units of $g^2 N_c/\pi$. $\text{P} \int dx$ denotes the principal value integral defined by

$$\text{P} \int dx f(x) \equiv \frac{1}{2} \int dx \left[f(x+i\epsilon) + f(x-i\epsilon) \right]_{\epsilon \rightarrow 0} \quad (1.5)$$

For comparison, we also list the Bethe–Salpeter equations for the fermion–fermion and the boson–boson bound states:

$$\begin{aligned} \mu_{(ff)}^2 \tilde{\varphi}_{(ff)}(x) &= \left(\frac{\alpha_{(f)1}}{x} + \frac{\alpha_{(f)2}}{1-x} \right) \tilde{\varphi}_{(ff)}(x) - \text{P} \int_0^1 \frac{dy}{(y-x)^2} \tilde{\varphi}_{(ff)}(y) \\ \mu_{(bb)}^2 \tilde{\varphi}_{(bb)}(x) &= \left(\frac{\alpha_{(b)1}}{x} + \frac{\alpha_{(b)2}}{1-x} \right) \tilde{\varphi}_{(bb)}(x) - \text{P} \int_0^1 \frac{dy}{(y-x)^2} \frac{(x+y)(2-x-y)}{4x(1-x)} \tilde{\varphi}_{(bb)}(y) \end{aligned} \quad (1.6)$$

It is important to note that while the expressions for the full propagators in (1.2) involve the infrared cutoff λ_- , the bound state equations (1.4), (1.6) are independent of this cutoff.

The bound state equations in (1.4), (1.6) are essentially Schrödinger equations $\mu^2 \varphi(x) = (H\varphi)(x)$. The “Hamiltonian” operator in the case of the boson–fermion (or the boson–boson) bound state equation on the right hand side is not Hermitean with respect to the standard measure $\int_0^1 dx$. We may conjugate by \sqrt{x} to define

$$\varphi_{(bf)}(x) \equiv \sqrt{x} \tilde{\varphi}_{(bf)}(x) \quad (1.7)$$

Then the bound state equation becomes Hermitean with respect to the standard measure;

$$\mu_{(bf)}^2 \varphi_{(bf)}(x) = \left(\frac{\alpha_{(b)1}}{x} + \frac{\alpha_{(f)2}}{1-x} \right) \varphi_{(bf)}(x) - \text{P} \int_0^1 \frac{dy}{(y-x)^2} \frac{(x+y)}{2\sqrt{xy}} \varphi_{(bf)}(y) \quad (1.8)$$

Application of the general arguments given in [2][6] show that the widths of these “meson” bound states are of higher order, of $\mathcal{O}(N_{\text{flavors}}/N_c)$ and the spectrum is real. The three point coupling between these mesons is of $\mathcal{O}(1/\sqrt{N_c})$.

The behavior of the bound state wave function at the boundaries may be obtained following [2]. If we denote the behavior of $\varphi_{(bf)}$ as

$$\varphi_{(bf)}(x) \stackrel{x \sim 0}{\sim} x^{\beta_{(b)1}} \quad \varphi_{(bf)}(x) \stackrel{x \sim 1}{\sim} (1-x)^{\beta_{(f)2}} \quad (1.9)$$

then the boundary behavior at $x = 0, 1$ is determined by the masses of the constituents from the equations

$$\alpha_{(b)1} - \pi\beta_{(b)1} \tan(\pi\beta_{(b)1}) = 0 \quad \alpha_{(f)2} + \pi\beta_{(f)2} \cot(\pi\beta_{(f)2}) = 0 \quad (1.10)$$

The boundary behavior at $x = 0$ and 1 is that of the boson–boson and fermion–fermion bound state wave functions respectively. Physical considerations restrict the boundary conditions to $\beta > 0$ except for the case $m_f^2 = 0$, when $\beta_{(f)} = 0$ is also allowed.

Approximate eigenstates may be obtained for the higher mass states as

$$\varphi_{(bf)k} \sim \sqrt{2} \sin(\pi k x) \quad (1.11)$$

which has the bound state mass of approximately $\pi^2 |k|$ leading to a linear trajectory asymptotically. This behavior is the same as that seen for the fermion–fermion and the boson–boson bound states.

The boson–fermion bound state equation is a natural generalization of the fermion–fermion, boson–boson bound state equations derived previously. In the large- N_c limit in two dimensions, the bound state is a “meson” formed out of a scalar or a spinor quark and an anti-quark in a linear confining potential. From the point of view of applying the results of this work to the standard model, the spectrum contains no massless fermions even when the original theory has chiral symmetry. This, in agreement with the results derived in other approaches [7], is unlike what is seen in the real world. However, to analyze the behavior of the strongly coupled standard model, we need to consider the effect of the scalar quartic coupling as well. We would also like to understand these bound states within the string picture of two-dimensional QCD [8].

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